Velocity and Speed

Key Ideas

- The average velocity of an object during a time period is the displacement of the object divided by the length of the time interval.
- The velocity of an object, sometimes called the instantaneous velocity, is the time derivative of the position, $\vec{x}(t)$. This is equal to the average velocity in the limit as the length of the time interval goes to zero.
- The velocity is a vector, giving the direction of motion.
- The speed of an object is the magnitude of the velocity, a positive scalar. The average speed of an object during a time interval is the distance traveled divided by the length of time.

Learning Objectives

By the end of this section, you should be able to:

- distinguish between the velocity at a specific time (the instantaneous velocity), the average velocity, and the speed,
- determine or estimate the average velocity of an object from a graph of position vs. time,
- determine the velocity of an object from the derivative of the position, $\vec{x}(t)$, and
- use a constant velocity approximation to solve motion problems for one or multiple objects.

In an era where we routinely use cars, trains, and airplanes, many of the concepts concerning motion are very familiar. We routinely experience a wide range of speeds, and the relationship between speed and distance traveled is a familiar concept; while an automobile may be an appropriate choice when driving to the neighboring town, an airplane may be more convenient for travel from New York to Los Angeles due to the time that is required. We have plenty of devices for measuring time and distance. Smartphones have apps for reporting GPS coordinates and tracking movement to help you navigate from one place to another. The app uses the built in distance data, traffic data, and the equations in this section to determine the time your trip will take.

The previous section introduced the concepts of position and displacement, or change in position. This section introduces the concept of **velocity**, which describes how position changes with time, for motion in one dimension (1D). This will use two concepts from calculus, derivatives and integrals. This requires that we introduce and use the concept of time. For the physics we will be considering in the first two volumes of this textbook, we'll consider **time** as a continually changing quantity that is measured by clocks and timers. Physical quantities such as position and velocity are continuous functions of time, $\vec{x}(t)$, $\vec{v}(t)$, respectively. Our understanding of time and space evolved in the Twentieth Century with the advent of Relativity Theory and Quantum Mechanics, but those effects do not affect our consideration here.

Average Velocity

Let's consider an object, maybe a person or a car, moving in one dimension. One example would be the professor walking to the right and left in front of a white board in the previous section.

At any time there is a position of the object, $\vec{x}(t)$, or using a coordinate system, the position can be described by the x-coordinate, $x(t)$. Considering two times, $t_1 < t_2$, the object's coordinates are $x_1 = x(t_1)$ and $x_2 = x(t_2)$ and the displacement is $\Delta x = x_2 - x_1$. The average velocity of the object during this time, $\Delta t = t_2 - t_1$, is equal to the displacement divided by the elapsed time. The average velocity is the ratio of the change in position to the change in time and has the dimensions of length over time.

AVERAGE VELOCITY

If x_1 and x_2 are the position coordinates in one dimension of an object at times t_1 and t_2, respectively, where $t_2 > t_1$, then the average velocity is

$$\begin{aligned} v_{ave} &= \frac{\text{displacement}}{\text{elapsed time}} \\ &= \frac{\Delta x}{\Delta t} \\ &= \frac{x_2 - x_1}{t_2 - t_1} \end{aligned} \tag{3.6}$$

The average velocity is a vector quantity. The SI units for velocity are m/s.

The velocity as indicated in Equation 3.6, can be either positive or negative if $x_2 > x_1$ or $x_2 < x_1$ respectively. If the object is moving along the x-axis and the velocity is negative, then the object is moving in the $-x$ direction or to the left. A positive velocity indicates motion in the $+x$ direction.

The expression for the average velocity can be used to solve problems where we need to determine changes in position or time intervals.

Example 3.2

How long will it take?

You drive south on a highway, starting from your house and going to visit a town $150\ \text{km}$ away. Along the way, you stop at a rest area that is $65\ \text{km}$ from your home for $15\ \text{minutes}$. While you are driving, your car is moving with an average velocity of magnitude $115\ \text{km/h}$, or about $40.0\ \text{m/s}$. (a) How long does it take for you to reach the rest area? (b) How long does it take for you to reach your final destination? (c) What is your average velocity for the entire trip?

Strategize

"How long" means we want to find the elapsed time of the trip The trip consists of three parts, from your home to the rest area, from the rest area to your final destination, and while you are at the rest area. Using the relation between average velocity, displacement, and the time interval, we can find the time you are driving.

We'll use a coordinate system with the origin at your home and the positive $\mathbf{x}$-direction as south, the direction you drive.

We need to be careful that we are using correct units, as the given values are in kilometers, meters, hours, minutes, and seconds.

Sketching a graph of your position versus time for the trip will help us understand and define the problem. We can use the average velocity for each time interval to picture the motion, as this will give the correct starting and ending points and time elapsed for each part of the trip.

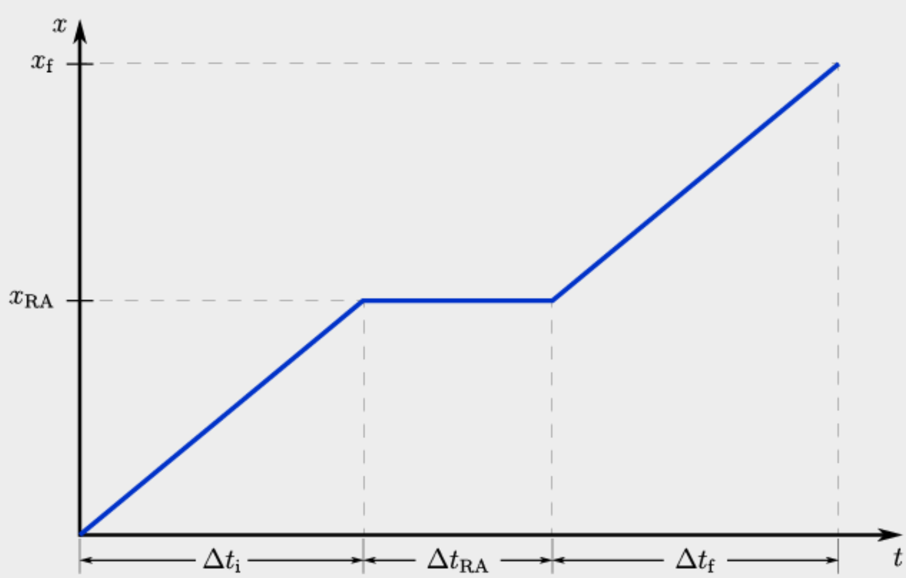

Figure 3.6 Position versus time for a car on a trip is shown, with a stop sometime in the middle. The average velocity of the car is used to correctly relate displacements and time intervals.

Develop and Solve

Use the graph above to identify the important properties of the problem.

Given values: $x_i = 0.0\ \text{km}, x_{RA} = 65.0\ \text{km}, x_f = 150.0\ \text{km},\ v_{ave} = 115\ \text{km/h},\ \text{and}\ \Delta t_{RA} = 15\ \text{min} = 0.25\ \text{h}$

Unknown quantities are the time intervals Δt_i (the time driving to the rest area), Δt_f (the time driving from the rest area to the destination), and the total time Δt_{Total}.

Equation 3.6 can be solved for each interval the car is moving for the elapsed time of that interval by dividing the displacement during that interval by the given average velocity.

$$\Delta t = \frac{\Delta x}{v_{ave}}$$

(a) For the first interval from home to the rest area, the elapsed time is

$$\begin{aligned}\Delta t_i &= \frac{\Delta x}{v_{ave}} \\ &= \frac{x_{RM} - 0}{v_{ave}} \\ &= \frac{65\ \text{km}}{115\ \text{km/h}} \\ &= 0.5652\ \text{h}\end{aligned}$$

Since there are sixty minutes in one hour, this is about 34 minutes. We've kept an extra digit because we'll be using this time interval for the calculation in parts (b) and (c) and wish to avoid round off error.

(b) After spending $0.25\ \text{h}$ at the rest stop, the elapsed time for the final part of the journey is found as we did in part (a).

$$\begin{aligned}\Delta t_f &= \frac{\Delta x}{v_{ave}} \\ &= \frac{x_f - x_{RA}}{v_{ave}} \\ &= \frac{150\ \text{km} - 65\ \text{km}}{115\ \text{km/h}} \\ &= 0.0.7391\ \text{h}\end{aligned}$$

The total elapsed time for the trip is the sum of the three time intervals.

$$\Delta t = 0.5652\ \text{h} + 0.25\ \text{h} + 0.7391\ \text{h} = 1.554\ \text{h}$$

(c) The average velocity for the entire trip is equal to the total displacement dividied by the total elapsed time for the three segments.

End of Content Preview.